Total No. of Questions—**12**] [Total No. of Printed Pages—**8**

Seat No.	

[5057]-61

S.E. (E&TC, Electronics) (Second Semester)

EXAMINATION, 2016

ENGINEERING MATHEMATICS-III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** from Section I and Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12** from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam table is allowed.

(*v*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $(D^3 + D^2 + D + 1)\, y = \cos^2 x$.

(*ii*) $(D^2 + 1)\, y = \dfrac{1}{1 + \sin x}$ (By variation of parameters)

(*iii*) $(2x + 1)^2 \dfrac{d^2y}{dx^2} - 2\,(2x + 1)\dfrac{dy}{dx} - 12\,y = 6$

(*iv*) $\dfrac{dx}{y + z} = \dfrac{dy}{-(x + z)} = \dfrac{dz}{x - y}$

(*b*) An electric circuit consists of an inductance L of 0.1 H, a resistance of R of 20 Ω and a condenser of capacitance C of 100 microfarads. If the differential equation of the electric circuit is : [5]

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

find charge q and current i at any time t.

[At $t = 0$, $q = 0.05$, $i = 0$]

Or

2. (*a*) Solve any *three* : [12]

(*i*) $(D^2 + 4)\, y = x \sin x$

(*ii*) $(D^2 + 6D + 9)\, y = \dfrac{e^{-3x}}{x^3}$

(*iii*) $(D^2 - 4D + 4)\, y = e^{2x} \sec^2 x$ (By variation of parameters)

(*iv*) $x^2 \dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = x^5$

(*b*) Solve : [5]

$$\frac{dx}{dt} + y = \sin t$$

$$\frac{dy}{dt} + x = \cos t$$

3. (*a*) If $f(z) = u + iv$ is an analytic function and $v = 3x^2y - y^3$, find u and express $f(z)$ in terms of z. [5]

(*b*) Evaluate : [5]

$$\oint_C \frac{e^{2z}}{z(z-1)^2}\, dz \quad \text{over} \quad |z| = 3.$$

(*c*) Find the bilinear transformation which maps the points $-1, 1, 0$ from z plane to the points $0, i, 3i$ of the w plane. [6]

Or

4. (*a*) Show that an analytic function with constant magnitude is constant. [5]

(*b*) Evaluate : [5]

$$\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$$

(*c*) Show that the bilinear transformation :

$$w = \frac{2z+3}{z-4}$$

maps the circle $x^2 + y^2 - 4x = 0$ onto the straight line $4u + 3 = 0$. [6]

5. (*a*) Using Fourier integral representation, show that : [6]

$$\int_0^\infty \frac{\sin \lambda\pi}{1-\lambda^2} \sin \lambda x \, d\lambda = \frac{\pi}{2} \sin x; \; 0 < x < \pi$$

(*b*) Find the Fourier cosine transform of $f(x) = x^{m-1}$. [5]

(*c*) Find the *z*-transform of the following (any *two*) : [6]

(*i*) $f(k) = c^k \cosh \alpha k; \; k \geq 0, \; c > 0$

(*ii*) $f(k) = ke^{-ak}, \; k \geq 0$

(*iii*) $f(k) = \frac{\sin ak}{k}, \; k > 0$

Or

6. (*a*) Find the inverse *z*-transform of the following (any *two*) : [6]

(*i*) $F(z) = \frac{z}{(z-1)(z-2)}; \; 1 < |z| < 2$

(*ii*) $F(z) = \frac{z(z+1)}{z^2 - 2z + 1}; |z| > 1$

(*iii*) $F(z) = \frac{1}{z-a}; |z| > |a|$

(*b*) Obtain $f(k)$ such that : [5]

$$f(k+1) + \frac{1}{2} f(k) = \left(\frac{1}{2}\right)^k; \; k \geq 0, f(0) = 0$$

(*c*) Solve the integral equation : [6]

$$\int_0^\infty f(x) \cos \lambda x \, dx = 1 - \lambda; \; 0 < \lambda < 1$$

SECTION II

7. (*a*) Find Lagrange's interpolating polynomial passing through the following points : [5]

x	y
0	4
1	3
2	6

Hence find y at $x = 1.5$.

(*b*) Use Simpson's $\frac{1}{3}$rd rule to obtain :

$$\int_0^{\pi/2} \frac{\sin x}{x}\, dx$$

by dividing the interval into four parts. [5]

(*c*) Use Runge-Kutta method of fourth order to obtain the numeric solution of the differential equation :

$$\frac{dy}{dx} = x^2 + y^2$$

with $y(1) = 1.5$ in the interval (1, 1.2) with $h = 0.1$ [6]

Or

8. (*a*) Evaluate : [5]

$$\int_0^1 \frac{dx}{1 + x^2}$$

taking $h = \frac{1}{6}$ using Simpson's $\frac{3}{8}$th rule.

(*b*) Use Euler's modified method to find y when $x = 0.1$ given that :

$$\frac{dy}{dx} = x^2 + y;\ y(0) = 1,$$

correct upto four decimal places. [5]

(*c*) With usual notations, establish : [6]

(*i*) $(1 + \Delta)(1 - \nabla) = 1$

(*ii*) $\left(1 + \frac{\delta^2}{4}\right)^{\frac{1}{2}} = \mu$

9. (*a*) Find the angle between the surfaces : [5]

$$x^2 + y^2 + z^2 = 9 \text{ and } z = x^2 + y^2 - 3$$

at the point (2, –1, 2).

(*b*) Find the directional derivative of :

$$\phi = x^2 - y^2 - 2z^2$$

at the point P(2, –1, 3) in the direction of $\overline{PQ}$, where Q is (5, 6, 4). [6]

(*c*) Show that the vector field : [6]

$$\overline{F} = (2xz^3 + 6y)\,\overline{i} + (6x - 2yz)\,\overline{j} + (3x^2z^2 - y^2)\,\overline{k}$$

is irrotational. Find the scalar potential ϕ such that $\nabla\phi = \overline{F}$.

Or

10. (*a*) If ϕ and ψ satisfy the Laplace equation, then prove that the vector : [5]

$$\overline{F} = \phi\nabla\psi - \psi\nabla\phi$$

is solenoidal.

(*b*) Prove that $\overline{F} = (\overline{a} \cdot \overline{r})\,\overline{a}$ is conservative. Find the scalar potential ϕ such that $\overline{F} = \nabla\phi$. [6]

(*c*) Prove the following (any *two*) : [6]

(*i*) $\nabla \times [\overline{a} \times (\overline{b} \times \overline{r}] = \overline{a} \times \overline{b}$ where $\overline{a}$ and $\overline{b}$ are constant vectors.

(*ii*) $\overline{b} \times \nabla\,[\overline{a} \cdot \nabla \log r] = \dfrac{\overline{b} \times \overline{a}}{r^2} - \dfrac{2(\overline{a} \cdot \overline{r})}{r^4}(\overline{b} \times \overline{r})$

(*iii*) $\nabla^2\left[\nabla \cdot \left(\dfrac{\overline{r}}{r^2}\right)\right] = \dfrac{2}{r^4}$

11. (*a*) Using Green's theorem evaluate : [6]

$$\oint_C [(3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy]$$

where C is the boundary of the region defined by $y^2 = x$ and $y = x^2$.

(*b*) Evaluate : [5]

$$\iint_S (2y\,\overline{i} + yz\overline{j} + 2\,xz\overline{k}) \cdot \hat{n}\,dS$$

over the surface of region bounded by :

$$y = 0,\ y = 3,\ x = 0,\ z = 0,\ x + 2z = 6$$

(*c*) Evaluate :

$$\iint_S \nabla \times \bar{F} \, . \, \hat{n} \; dS$$

by Stokes' theorem where S is the surface bounded by the upper half of $x^2 + y^2 + z^2 = 1$ and its projection on the xy plane. [6]

$$\bar{F} = (2x - y)\,\bar{i} - yz^2\,\bar{j} - y^2\, z\bar{k}.$$

Or

12. (*a*) Find the work done in moving a particle along a straight line from O (0, 0, 0) to A (2, 1, 3) in force field : [6]

$$\bar{F} = 3x^2\,\bar{i} + (2x\,z - y)\,\bar{j} + z\bar{k}.$$

(*b*) Show that : [5]

$$\iiint_V \frac{dv}{r^2} = \iint_S \frac{\bar{r} \, . \, \hat{n}}{r^2} \, dS$$

(*c*) Apply Stokes' theorem to evaluate : [6]

$$\oint_C (4y\,dx + 2z\;dy + 6y\;dz)$$

where C is the curve of intersection of :

$$x^2 + y^2 + z^2 = 6z \quad \text{and} \quad z = x + 3.$$